Available online at www.sciencedirect.com
SCIENCE DIRECT®

ELSEVIER

Journal of Sound and Vibration 273 (2004) 317–335

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Feedback controller design for sensitivity-based damage localization

B.H. Koh*, L.R. Ray

Thayer School of Engineering, Dartmouth College, 8000 Cummings Hall, Hanover, NH 03755, USA

Received 19 July 2002; accepted 16 April 2003

Abstract

A method is developed for locating structural damage using only measured natural frequency changes induced by damage. The damage localization method exploits multiple sensitivity enhancing controllers, each of which provides an independent set of modal frequency information that is used to identify damage variables based on a least-squares technique. The method provides significant improvement in damage localization and ability to tolerate measurement noise on natural frequency shifts due to damage over similar localization methods that use only open-loop modal data. A first order sensitivity matrix relates natural frequency changes in both the open- and closed-loop systems to damage variables and is evaluated from an analytic model. Single- and multi-input control laws are designed to enhance the change in natural frequencies due to damage. Multi-input control laws are designed using a minimum-gain eigenstructure assignment method in order to maintain a well-conditioned sensitivity matrix and generate independent modal data, while also minimizing the number of actuators required. This study found that the resolution of measured natural frequency changes due to damage can be significantly improved by careful selection of damage-sensitive closed-loop poles targeted by the eigenstructure assignment method. As a result, measured closed-loop natural frequency changes due to damage exhibit better signal-to-noise ratios than open-loop frequency changes. The method is demonstrated numerically using a cantilevered beam to show how multiple sensitivity enhancing controllers can locate damage and assess damage extent in the presence of measurement noise on natural frequencies.

1. Introduction

Increasing performance demands on load-carrying structures, along with high expected reliability, have necessitated the development of elaborate non-destructive evaluation (NDE)

*Corresponding author. Tel.: +1-603-646-1243; fax: +1-603-646-3856.
E-mail address: Bong-Hwan.Koh.Adv03@alum.dartmouth.org (B.H. Koh).

0022-460X/03/$ - see front matter
doi:10.1016/S0022-460X(03)00541-8

methods. Among them, vibration-based damage detection methods have received attention due to their simplicity and autonomous monitoring capability.

Natural frequency (eigenvalue) and mode shape (eigenvector) changes have been the most frequently studied vibration-based damage metrics [1]. Inverse or model updating approaches to damage identification use optimization methods to update mass and stiffness matrices of an analytic model using measured modal data. Using the structure's eigenvalue equation for damage localization requires both eigenvalue and eigenvector measurements to reconstruct stiffness and mass parameters. This means that perturbation or damage resulting in changes in system parameters cannot be determined explicitly without knowing the perturbation in both eigenvalues and eigenvectors simultaneously. However, mode shape measurement requires multiple sensors to approximate the infinite number of degrees of freedom (d.o.f.) in real distributed parameter structures. When mode shape measurements are incomplete, mode expansion or model reduction inevitably deteriorates the solution of the eigenvalue equation through inverse approaches [2,3]. Moreover, measurement error and noise are critical threats to the credibility of measured mode shape information [4]. On the other hand, modal frequencies (eigenvalues) give more reliable evidence of damage in a structure, compared to the mode shape (eigenvector), in that frequencies are easier to measure and are less sensitive to measurement errors [5]. Hence, damage detection methods that require only measured modal frequencies are generally preferred to those that require both frequencies and mode shapes. Nonetheless, when using modal frequencies for damage localization, the relationship between a finite set of measured modal frequencies and damage location is not guaranteed to be unique.

The model updating method can be cast into a form in which only frequencies are required for damage localization, when damage variables can be posed a priori. Sensitivity-based model updating methods use the first order approximation between system parameter perturbations and modal frequency changes to determine damage parameters using a least-squares approach. For example, Messina et al. [6] propose a sensitivity-based method which is combined with statistical correlation to find multiple damage locations in truss and three-beam test structures. After inserting the first order sensitivity matrix into a correlation equation, the damage scaling coefficients are sought, which avoids taking a pseudo-inverse of the sensitivity matrix. A numerical simulation study is presented in Ref. [6] to validate the method. In the study, more than 10 measured modal frequencies are required for detecting multiple damage locations. The effectiveness of a second order approximation is also investigated. Usually, the sensitivity matrix is obtained using an analytic model by finding derivatives of the eigenvalues with respect to stiffness perturbations for each potential damage location. The updating process typically iterates the solution until the damage variables satisfactorily converge to the true system perturbations. One problem with this method, which is again, related to mathematical uniqueness of damage localization using modal frequencies, is that the number of unknowns (system parameters to be updated) is usually much larger than the number of measurable modal frequencies. In other words, the number of identifiable damage locations is limited to the number of measurable modes. It is quite difficult to accurately measure more than a few modal frequencies above the lowest one. As a result, recent research has focused on methods of increasing the modal frequency data available for sensitivity-based model updating in order to enhance uniqueness characteristics.

Researchers have proposed a variety of methods for enriching the modal data set in order to enhance model updating. Cha and Gu [7] studied a mass addition technique for structural parameter updating. They show that adding known masses to a multi-spring–mass system and measuring its new eigendata can correct the mass matrix of the perturbed model. Subsequently, the stiffness matrix can also be updated by equating the eigenvalue equation to a new, mass-added system. However, this method needs both natural frequency and mode shape measurements. Nalitolela et al. [8] introduce the possibility of using a mass or stiffness added model to extract additional natural frequencies. After perturbing mass or stiffness at specific co-ordinates of the beam, a sensitivity analysis is performed. In order to measure significant natural frequency changes, substantial perturbation of the structure is essential. Physically adding mass or stiffness to structures makes the method difficult to implement in practice. Lew and Juang [9] incorporate the concept of a virtual passive controller to overcome this limitation. They used output and dynamic feedback controllers to generate additional closed-loop modal frequencies in order to identify multiple damage locations in a cantilevered beam. Here, no physical mass or stiffness attachments to the structures are required. In the study, a damage variable vector is defined as a percentage of stiffness loss for each potential damage location. Although the stability of the closed-loop system is guaranteed by the nature of the energy dissipative passive controller, neither the sensitivity of frequency shifts for each passive controller nor the effect of measurement noise is considered. In general, frequency shifts due to small damage in structures are insignificant [5,10]. Hence, without sensitivity enhancement, it is difficult to measure modal frequency changes accurately in the presence of measurement noise. Recently, Palacz and Krawczuk [11] investigate the effects of measurement error on damage detection using vibration parameters. The study shows that very small errors in measured natural frequencies can ruin the localization of damage in a cantilevered beam. Thus, to guarantee that additional modal data provides linearly independent equations for sensitivity-based damage localization, control laws must be properly designed.

Ray and Tian [12] propose a new approach to improve sensitivity of closed-loop natural frequency toward stiffness and mass damage. They employ a damage-sensitive pole placement technique. In that study, feedback control laws for sensitivity enhancement are investigated through numerical simulation in order to detect damage in a cantilevered beam. A control law targeting stiffness damage detection is designed by reducing frequencies of the first three modes using a single point force actuator. Sensitivity to the thickness reduction at the root of the beam increases by a factor of approximately 40 (first mode) to a factor of five (third mode). Ray et al. [13] extends these results to consider sensitivity enhancement of fatigue crack damage and experimental validation of the SEC concept for a cantilevered beam under bending.

Both Refs. [12,13] use sensitivity enhancing control law designs for single-input systems to increase the linear sensitivity of natural frequency variations to damage. For a completely controllable system, the control gains are uniquely determined in the single-input case given a set of closed-loop pole locations. Key to the development of the SEC concept for single-input systems is a methodology for control law design in order to maximize sensitivity enhancement. Ray and Marini [14] develop an optimization method for single-input systems in which optimal closed-loop pole locations are determined for SEC of mass or stiffness damage, given constraints on control effort and fixed actuator location.

In the multi-input case, an infinite number of gain sets can satisfy closed-loop pole placement. Hence, additional control law design objectives can be achieved in the multi-input case. Juang et al. [15] propose an eigenstructure assignment technique seeking a minimum norm gain solution for an output feedback multi-input system. In that study, a null-space technique along with singular value decomposition is adopted to expand admissible eigenvector space. They suggest the open-loop eigenvector as a good candidate for a desired closed-loop eigenvector set, which eventually leads to minimum control gains and thus minimum control effort.

Both Refs. [12,13] focus on damage detection without regard to location or damage extent. The key contribution of this paper is developing and demonstrating the use of the SEC concept introduced in Refs. [12,13] for creating damage sensitive feedback controllers in order to provide sufficient and independent equations to solve for unknown damage parameters or damage locations. The sensitivity-based damage localization method of Ref. [6] is combined with the concept of damage-sensitive control laws of Refs. [12,13] to produce a damage localization method that relies on fewer measured modal frequencies than a comparable open-loop system only and is robust to measurement noise. The eigenstructure assignment method of Ref. [15] is used to construct multi-input systems for SEC. Concepts from Ray and Marini [14] are used to identify closed-loop pole locations that maximize sensitivity for fixed actuator locations. Multiple actuators along a structure can contribute additional sets of measured closed-loop modal frequencies in order to maintain a sensitivity matrix relating damage variables and measured frequencies that is well-conditioned, and thus, whose pseudo-inverse is defined. All possible actuator locations are investigated to increase the orthogonality of the equations. An eigenstructure assignment technique is presented to determine the minimum norm gains for multi-input closed-loop systems. In essence, damage-sensitive control laws improve the resolution of measured natural frequency changes.

A simulation example is presented that demonstrates the effect of additional closed-loop modal information through single-input and multi-input controllers on the numerical conditioning of sensitivity-based damage localization. Through this example, it is demonstrated that the condition number alone is not an accurate estimate of damage localization ability. Scalar metrics that can be used to compare suitability of various actuator-location and control law combinations for enhancing damage localization are presented.

2. Background theory

2.1. Sensitivity enhancing control

A damage-sensitive feedback controller drives the poles of the closed-loop system to the locations in the complex plane where modal frequency shifts are more sensitive to damage [12,13]. The concept of sensitivity enhancement can be easily demonstrated by numerical simulation of a controllable structural model. For example, the FE cantilevered beam model considered here consists of 8 elements and 9 nodes with a point force actuator at node 2 as shown in Fig. 1. Single-input pole placement (SIPP) is employed to move the first four modes to frequencies slightly lower than open-loop frequencies in the complex plane. Damage-sensitive feedback control increases the frequency shifts under 10% reduction of Young's modulus in the first element (closest to the root) of the beam. The percentage of frequency shifts in the first four modes of the closed-loop system are shown in Fig. 2 as function of closed-loop frequency. As the closed-loop frequency of each mode decreases, the frequency shifts increase, illustrating sensitivity enhancement.

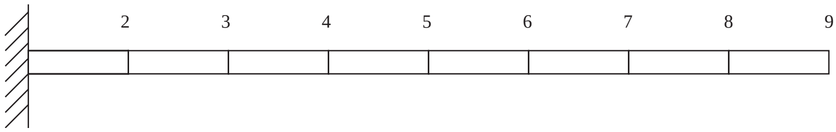

Fig. 1. Eight-element controlled cantilever beam finite element model and nodal points: 200 mm (W) $\times$ 9.5 mm (H) $\times$ 650 mm (L).

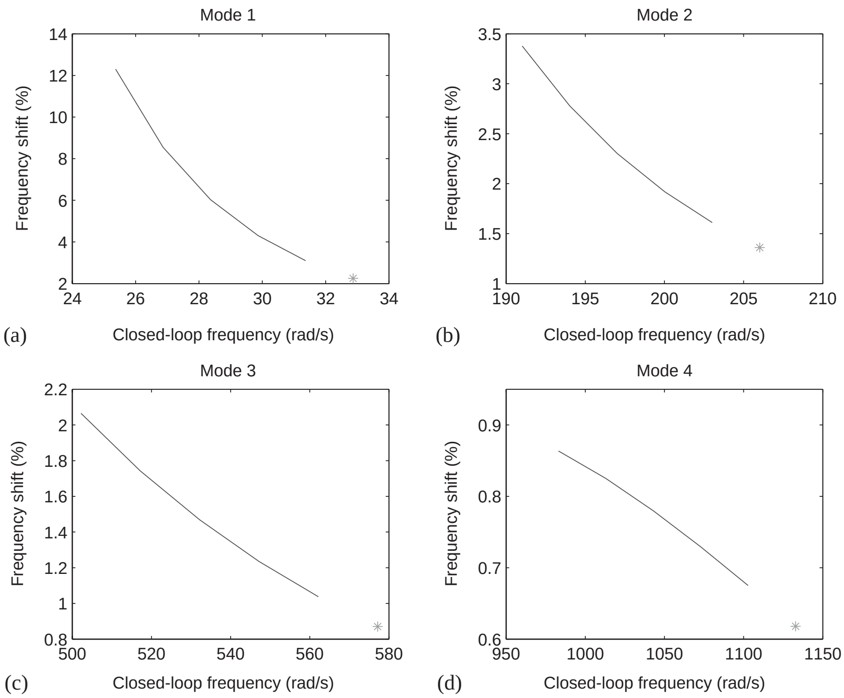

Fig. 2. Sensitivity enhancement in an 8-element cantilever beam: the percentage of natural frequency changes of the first four modes vs. closed-loop pole selections; (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4; $*$, denotes open-loop frequency.

2.2. Sensitivity-based damage localization

Damage localization using eigenvalue sensitivity analysis requires two consecutive processes: (1) computing the eigenvalue sensitivity matrix from an analytic model, and (2) estimating the unknown parameters or damage variables using natural frequencies measured from a real structure. The sensitivity matrix is typically computed from a finite element (FE) model of the structure. With this sensitivity matrix and measured natural frequencies, damage variables, i.e., possible locations of damage occurrence and their extent, should be identified. Here, the damage variable is defined as the fraction of thickness reduction for each finite element. For example, a damage vector $\{v_d\} = \{1, 1, 0.95, 1, 1, 1, 1, 1\}$ means that the third element has a 5% thickness reduction while the other seven elements are undamaged. The damage vector $\{v_h\}$, whose elements are all equal to unity, represents the nominal or healthy system.

The analytic model of a distributed parameter structure such as a cantilevered beam can be conveniently expressed by a second order equation as

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = \mathbf{B}_f u, \tag{1}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ and $\mathbf{B}_f$ are mass, damping, stiffness and influence matrices, respectively. For an equivalent control model, a state-space equation is defined as

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u, \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{B}_f \end{bmatrix}, \tag{3}$$

$$\mathbf{X} = [x \quad \dot{x}]^{\mathrm{T}}. \tag{4}$$

Denoting $\mathbf{A}_d$ as a damaged system matrix (by the perturbation of damage variables), the solution of the characteristic equation yields the ith natural frequency of damaged system:

$$\det[\mathbf{A}_d - (\omega_d)_i^2 \mathbf{I}] = 0. \tag{5}$$

Perturbations of the damage variable $\{\delta v\}$ and the system's natural frequency changes $\{\delta\omega\}$ are defined as

$$\{\delta v\} = \{v_h\} - \{v_d\}, \quad \{\delta\omega\} = \{\omega_h\} - \{\omega_d\}, \tag{6, 7}$$

where ω_h and ω_d represent the natural frequencies of the healthy and damaged structures, respectively.

The non-linear relation between $\{\delta\omega\}$ and $\{\delta v\}$ can be linearized by the first order multi-variable approximation:

$$\{\delta\omega\} = \mathbf{S}\{\delta v\}, \tag{8}$$

where

$$\mathbf{S} = \begin{bmatrix} \dfrac{\partial\omega^1}{\partial v_1} & \dfrac{\partial\omega^1}{\partial v_2} & \cdots & \dfrac{\partial\omega^1}{\partial v_r} \\ \dfrac{\partial\omega^2}{\partial v_1} & \dfrac{\partial\omega^2}{\partial v_2} & \cdots & \dfrac{\partial\omega^2}{\partial v_r} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial\omega^p}{\partial v_1} & \dfrac{\partial\omega^p}{\partial v_2} & \cdots & \dfrac{\partial\omega^p}{\partial v_r} \end{bmatrix}. \tag{9}$$

The sensitivity matrix, $\mathbf{S}$, is defined as derivatives of natural frequencies ω_i up to p modes with respect to r damage variables v_j. Hence, the size of the $\mathbf{S}$ matrix is $p \times r$.

Once the sensitivity matrix is formed, the actual damage variables can be estimated from the measured natural frequency shifts $\{\delta\omega\}_m$ and the pseudo-inverse of sensitivity matrix:

$$\{v_d\} = \{v_h\} - \mathbf{S}^{+}\{\delta\omega\}_m. \tag{10}$$

In this study, the damage variables are found directly from Eq. (10) with no iteration, thus diagnosis of greater degrees of damage may be more difficult due to the non-linear relationship between natural frequencies and damage level. Iterative updating using more than one set of measured modal frequencies could improve the converged solution of Eq. (10) as in Ref. [9], though its success also depends heavily on the accuracy of the analytic model.

The accuracy of the estimated damage variables $\{v_d\}$ depends on the condition number of sensitivity matrix, $\mathbf{S}$. When the number of measurable natural frequencies is less than the number of unknown damage variables ($p<r$), additional sets of closed-loop natural frequencies can be included to identify the damage variables. However, added closed-loop natural frequency vectors should be mutually independent. In other words, the condition number, or the ratio of the largest to smallest singular value of the sensitivity matrix should be as small as possible. Otherwise, the sensitivity matrix can become ill-conditioned and computing the pseudo-inverse, $\mathbf{S}^+$ might not be possible. Hence, the condition number of the sensitivity matrix is a critical measure of the adequacy of additional closed-loop information.

Given a controllable system model $(\mathbf{A}, \mathbf{B})$, the ith natural frequency of the closed-loop system formed by applying control law $u = -\mathbf{GX}$ to the actual system is obtained by the solutions to

$$\det[(\mathbf{A} - \mathbf{BG}) - (\omega^c)_i^2 \mathbf{I}] = 0. \tag{11}$$

As mentioned in Refs. [12,13], by choosing the gain matrix $\mathbf{G}$ properly, closed-loop poles can be arbitrary placed in the complex plane. Conversely, assigning the desired closed-loop pole locations provides a state feedback control gain. Thus, state feedback closed-loop systems will not only provide additional information for Eq. (8), but can also enhance the sensitivity of the natural frequency change by systematic selection of closed-loop poles. As a result, a new sensitivity matrix having p modes of closed-loop natural frequencies from q closed-loop systems is calculated as

$$\mathbf{S}^c = \begin{bmatrix} \dfrac{\partial \omega_1^{c1}}{\partial v_1} & \dfrac{\partial \omega_1^{c1}}{\partial v_2} & \cdots & \dfrac{\partial \omega_1^{c1}}{\partial v_r} \\ \dfrac{\partial \omega_2^{c1}}{\partial v_1} & \dfrac{\partial \omega_2^{c1}}{\partial v_2} & \cdots & \dfrac{\partial \omega_2^{c1}}{\partial v_r} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial \omega_p^{c1}}{\partial v_1} & \dfrac{\partial \omega_p^{c1}}{\partial v_2} & \cdots & \dfrac{\partial \omega_p^{c1}}{\partial v_r} \\ \dfrac{\partial \omega_1^{c2}}{\partial v_1} & \dfrac{\partial \omega_1^{c2}}{\partial v_2} & \cdots & \dfrac{\partial \omega_1^{c2}}{\partial v_r} \\ \dfrac{\partial \omega_2^{c2}}{\partial v_1} & \dfrac{\partial \omega_2^{c2}}{\partial v_2} & \cdots & \dfrac{\partial \omega_2^{c2}}{\partial v_r} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial \omega_p^{cq}}{\partial v_1} & \dfrac{\partial \omega_p^{cq}}{\partial v_2} & \cdots & \dfrac{\partial \omega_p^{cq}}{\partial v_r} \end{bmatrix}. \tag{12}$$

Here, ω_p^{cq} means pth natural frequency of the qth closed-loop system. As in Eq. (10), damage variables can be identified using the closed-loop sensitivity matrix, $\mathbf{S}^c$, which is computed from the analytic model, and measured closed-loop natural frequency shifts, $\{\delta\omega^c\}_m$:

$$\{v_d\} = \{v_h\} - \mathbf{S}^{c+}\{\delta\omega^c\}_m. \tag{13}$$

Note that the number of rows in $\mathbf{S}^c$ $(pq \times r)$ increases by a factor of q, compared to the open-loop sensitivity matrix $\mathbf{S}$ $(p \times r)$. Although, theoretically, the number of damage variables $\{v\}$ can be increased up to the number of rows (pq) of $\mathbf{S}^c$, the unique solution of Eq. (13) is not guaranteed unless the sensitivity matrix is well conditioned. Hence, the sets of closed-loop systems should be chosen such that the condition number of sensitivity matrix is as small as possible, subject to constraints in the number of actuators and the maximum control effort. In this study, each closed-loop system has a different actuator location, which improves the independence of the rows of $\mathbf{S}^c$ and thus minimizes the condition number.

2.3. Multi-input controller design

Previous studies [12–14] on damage-sensitive feedback control investigate only single-input closed-loop systems. In particular, Ref. [14] considers optimal closed-loop pole locations for SEC using single-input system. In this section, the multi-input case is investigated from the viewpoint of damage-sensitive controller design for damage localization.

First, a general approach to the determination of feedback gain matrices from Ref. [16] is summarized. The eigenvalue equation for a closed-loop system can be written as

$$(\mathbf{A} + \mathbf{BG})\boldsymbol{\Psi}_k = \boldsymbol{\Psi}_k\lambda_k, \tag{14}$$

where $\boldsymbol{\Psi}_k$ represents an kth eigenvector corresponding to the assigned or desired closed-loop eigenvalue λ_k. Alternatively, Eq. (14) can be partitioned in a homogeneous form as

$$[\mathbf{A} - \lambda_k\mathbf{I} \,|\, \mathbf{B}]\begin{bmatrix} \boldsymbol{\Psi}_k \\ \mathbf{G}\boldsymbol{\Psi}_k \end{bmatrix} = 0. \tag{15}$$

Eq. (15) should have a non-trivial solution to satisfy the sufficient condition for the existence of assigned eigenvalues and their corresponding eigenvectors.

Defining

$$\boldsymbol{\Gamma}_k = [\mathbf{A} - \lambda_k\mathbf{I} \,|\, \mathbf{B}]. \tag{16}$$

Singular-value decomposition (SVD) can be used to find a set of orthogonal basis vectors spanning the null space of Eq. (16):

$$\boldsymbol{\Gamma}_k = \mathbf{U}_k\boldsymbol{\Sigma}_k\mathbf{V}_k^* = \mathbf{U}_k\begin{bmatrix} \sigma_k & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{V}_{\sigma k}^* \\ \mathbf{V}_{0k}^* \end{bmatrix}. \tag{17}$$

Thus, Eq. (17) gives

$$\boldsymbol{\Gamma}_k\mathbf{V}_{0k} = 0, \quad \boldsymbol{\Gamma}_k\phi_k = 0, \tag{18}$$

where

$$\phi_k = \begin{bmatrix} \boldsymbol{\Psi}_k \\ \tilde{\boldsymbol{\Psi}}_k \end{bmatrix}. \tag{19}$$

From Eqs. (15) and (19), the gain matrix $\mathbf{G}$ is embedded in $\tilde{\boldsymbol{\Psi}}_k$. Thus $\mathbf{G}$ can be obtained by taking the inverse of matrix $\boldsymbol{\Psi}_k$, the assigned eigenvectors, which are selected by the designer in advance:

$$\mathbf{G} = \tilde{\boldsymbol{\Psi}}_k \boldsymbol{\Psi}_k^{+}. \tag{20}$$

Since an infinite number of eigenvector and eigenvalue combination can be assigned in Eq. (14), usually additional constraints are imposed, such as minimizing the norm of the gain matrix or improving robustness toward system uncertainties. In this study, a gain-minimization technique from the study of Juang et al. [15] is applied to design the multi-input controller. Eventually, these multi-input closed-loop systems are considered to be additional candidates for building the sensitivity matrix of Eq. (12).

3. Simulations and results

For numerical simulations, an aluminum cantilevered beam is modelled using 64 uniform finite elements. The dimensions of the beam are 200 mm $(W) \times 9.5$ mm $(H) \times 650$ mm (L). This represents the truth model of the structure, which is used to generate damage cases and to which state-feedback control laws are applied. The state-feedback control laws for generating closed-loop systems are developed from an 8-element cantilevered beam model (Fig. 1) that has the same dimension of the 64-element model. Hence, the location for each node of 8-element model coincides with a corresponding node of the 64-element model.

The open-loop poles of the first three modes are -0.67 ± 115.77i, -26.3 ± 725.08i, and -206.6 ± 2022.1i, respectively. For sensitivity enhancement, the damped natural frequencies of the first three modes are lowered from open-loop poles such as (A) as shown in Table 1. For comparison, another set of closed-loop poles, case (B) is considered whose real parts of eigenvalues are increased, respectively. The reason for comparing these two cases is to show that the locations of closed-loop poles in the complex plane explicitly govern the amount of sensitivity enhancement, as illustrated in Section 2.1, and consequently, also affect damage localization. Thus, case (A) increases sensitivity to stiffness damage, while case (B) decreases sensitivity.

The actuator for the state-feedback closed-loop system is a vertical force input applied at a nodal point. Table 1 provides target closed-loop pole locations for five single-input control laws, with actuator locations at nodes 2, 3, 4, 6, and 7, and for five two-input control laws. Following Ref. [14], pole locations for single-input control laws are designed such that sensitivity enhancement is maximized, yet each control law requires comparable control effort. Double digits for actuator location indicate multi-input cases; for example, actuator location 23 represents a multi-input closed-loop system having dual actuators at nodes 2 and 3. Dual actuators allow a significant increase in the total number of independent controllers without increasing the number of actuators.

Stiffness damage is assumed as the damage case and is simulated as a 3% thickness reduction in a single element. Damage variables are selected as a set of potential single-element damage locations in the truth model. Three different damage variable vectors are considered as shown in Table 2. A damage vector having 11 damage variables can identify 11 potential damaged elements from total 64 elements, while one with 16 damage variables can identify 16 possible single-element damage locations.

Table 1
Actuator locations and assigned closed-loop poles for controllers (A) and (B)

Actuator locations	Closed-loop poles (A)	Closed-loop poles (B)
2	$-0.7\pm60i$ $-27.0\pm555i$ $-206.0\pm1800i$	$-1.0\pm115.77i$ $-36.0\pm725.08i$ $-255\pm2022i$
3	$-0.7\pm65i$ $-27.0\pm565i$ $-206.0\pm1750i$	$-1.2\pm115.77i$ $-46.0\pm725.08i$ $-285\pm2022i$
4	$-0.7\pm70i$ $-27.0\pm575i$ $-206.0\pm1700i$	$-0.9\pm115.77i$ $-40.0\pm725.08i$ $-275\pm2022i$
6	$-0.7\pm80i$ $-27.0\pm625i$ $-206.0\pm1700i$	$-1.5\pm115.77i$ $-30.0\pm725.08i$ $-235\pm2022i$
7	$-0.7\pm85i$ $-27.0\pm650i$ $-206.0\pm1750i$	$-2.1\pm115.77i$ $-33.0\pm725.08i$ $-225\pm2022i$
23	Same as 2	Same as 2
24	Same as 3	Same as 3
26	$-0.7\pm95i$ $-27.0\pm655i$ $-206.0\pm1700i$	$-2.7\pm115.77i$ $-50.0\pm725.08i$ $-225\pm2022i$
27	Same as 6	Same as 6
67	Same as 6	Same as 6

Table 2
Number of damage variables and corresponding element number of damage locations

No. of damage variables	Element number
11	1 7 13 19 25 31 37 43 49 55 61
13	1 6 11 16 21 26 31 36 41 46 51 56 61
16	1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65

Fig. 3(a) presents the percent change in open-loop natural frequencies for the first three modes due to thickness reduction in a single beam element of the 64-element truth model. It shows that the maximum open-loop frequency shifts are less than or equal to 0.3%, as

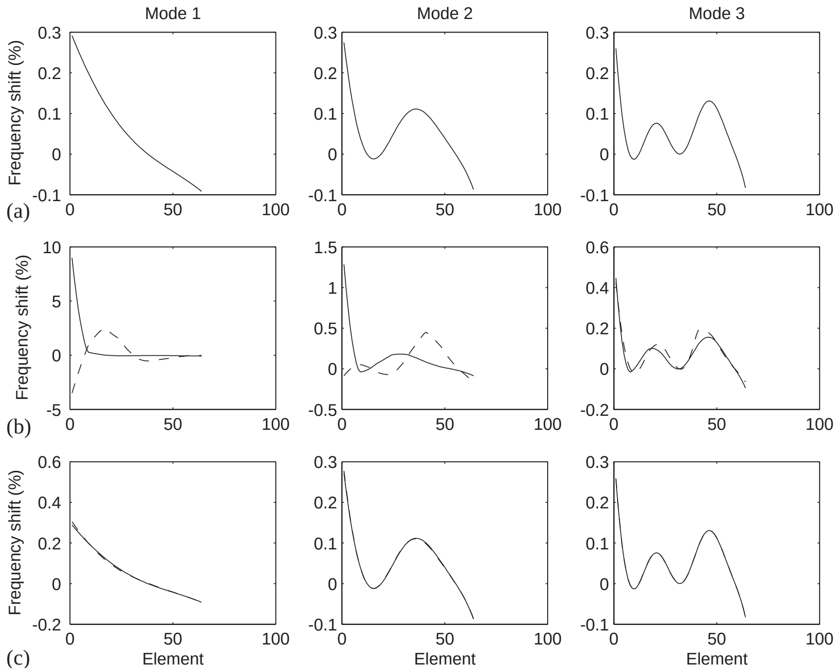

Fig. 3. Percentage frequency changes for damage consisting of a 3% thickness reduction in a single beam element of the 64-element model; (a) open-loop system, (b) closed-loop system (A), (c) closed-loop system (B). Single-input actuators at node 2 (–) and 6 (- - -) for closed-loop systems.

damage location moves from the root to the tip of beam. Figs. 3(b) and (c) illustrate the percent natural frequency shifts for closed-loop systems (A) and (B), having respectively, both single-input actuators at nodes 2 and 6. In contrast to the open-loop case, the closed-loop system with damage-sensitive controller (A) exhibits a huge enhancement in frequency shifts for the first three controlled modes. Note that damage-insensitive controller (B) does not show much difference from the open-loop case in frequency changes. It is also clear from Fig. 3(b) that under sensitivity enhancing control, the actuator location (nodes 2 and 6) can significantly change the *pattern* of frequency shifts due to damage along the beam. This contrast improves the independence of each closed-loop system's contribution to the sensitivity matrix of Eq. (12).

Table 3 presents damage localization results for several combinations of closed-loop systems which have p modes, q actuators, and r damage variables along with the resulting sensitivity matrix condition numbers. Given a fixed number of actuators and damage locations, condition number is a good measure of damage localization ability. However, condition number is generally not a good localization metric when comparing systems with a different number of actuators, or damage cases. In order to develop a scalar metric that adequately captures the effect of number of controllers and number of damage cases on localization, a quantitative measure of the localization performance is provided by two statistical indices, α and β. These indices indicate accuracy of localization and severity of damage, respectively.

Table 3
Number of measured modal frequencies (p), actuators (q), damage variables (r), condition numbers (CN) for closed-loop systems (A, B), and the result of damage localization (α/β) with no measurement noise

p	q	r	CN (A)	α/β (A)	CN (B)	α/β (B)
3	5	11	268	0.0047/0.0019	2118	0.0074/0.0130
3	5	13	1763	0.0258/0.0097	2912	0.0119/0.0175
3	5	16	4092$^{(9)}$	0.0757/0.0342	4693	6.1395/3.6884
4	5	11	167$^{(4)}$	0.0012/0.0019	2859$^{(5)}$	0.0071/0.0120
4	5	13	901	0.0098/0.0048	4354	0.0103/0.0145
4	5	16	2613	0.0249/0.0219	7748	0.0106/0.0156
3	10	11	275	0.0038/0.0019	1055	0.0066/0.0155
3	10	13	769	0.0061/0.0047	1674	0.0075/0.0175
3	10	16	1478$^{(10)}$	0.0150/0.0133	2118	0.0090/0.0197
4	10	11	188	0.0011/0.0018	1838	0.0061/0.0132
4	10	13	684	0.0055/0.0029	3064	0.0072/0.0154
4	10	16	1434	0.0072/0.0052	3645	0.0082/0.0175

($\cdot$) indicates a figure number of damage localization result. Five actuators at nodes (2, 3, 4, 6, 7), 10 actuators at nodes (2, 3, 4, 6, 7, 23, 24, 26, 27, 67).

$$\alpha = \bar{s}, \quad s_k = \sqrt{\sum_{\substack{i=1\\ i\neq k}}^{r} \frac{(v_i - \bar{v})^2}{r-2}}, \tag{21}$$

$$\beta = \bar{\gamma}, \quad \gamma_k = \frac{v_k - v_t}{v_t}. \tag{22}$$

α represents the mean of standard deviations (s_k) of total $r-1$ damage variables which are identified as healthy ones, i.e., $i \neq k$ (where k is the correct location of damage). β indicates the mean of ratios (γ_k) between the true value of a damage variable ($v_t = 0.97$) and the identified value of the kth damage variable (v_k). Hence, the smaller α and β, the better damage localization and the more accurate assessment of its severity.

There are three important observations to be made regarding Table 3. First, the values of α and β decrease as the condition numbers decrease. Showing that, the performance of damage localization depends on the condition number of the sensitivity matrix, $\mathbf{S}^c$. However, α and β provide more objective measures for localization performance than condition numbers since comparing the absolute values of condition numbers for each closed-loop system is less meaningful, due to variation in the number of and placement of actuators. In other words, one cannot compare condition numbers of systems which have dissimilar combinations of closed-loop systems. For example, in Table 3, closed-loop system (A) with $\{p = 3, q = 10, r = 11\}$ shows a condition number that is bigger than the case of $\{p = 3, q = 5, r = 11\}$, yet the $q = 10$ case still exhibits better damage localization result in terms of performance indices α and β. Likewise, the results from the case of $\{p = 4, q = 10, r = 11\}$ and $\{p = 4, q = 5, r = 11\}$ show the same pattern. Secondly, as the value of the number of damage variables r approaches pq, the condition number of the sensitivity matrix increases and damage localization becomes less successful. This supports the fact that simply satisfying the inequality, $pq > r$, does not guarantee successful localization [8]. Finally, in regard to sensitivity enhancement, controller (A) apparently gives better damage localization results than controller (B), and the difference in the damage severity index, β, between two controllers is substantial. Hence, sensitivity enhancement improves the performance of damage localization.

3.1. Measurement noise

Figs. 4–7 illustrate the result of damage localization using the closed-loop sensitivity matrix, $\mathbf{S}^c$. Each plot in the figure represents localization results for 11 damage variables along the beam. The vertical line of each plot denotes the true location of the damaged element. Abscissa and ordinate indicate the serial number of damage variables (Table 2) and their identified values, respectively.

First, the effect of measurement noise is examined. With the same noise level, damage localization using the sensitivity enhanced closed-loop systems (A) is compared to the case of closed-loop systems without sensitivity enhancement (B). Figs. 4 and 5 indicate localization results using controllers (A) and (B), respectively, and noise-free frequency measurements. Both results are from a sensitivity matrix, $\mathbf{S}^c$, developed for four modes (p), five closed-loop systems (q), and 11 damage variables (r) (i.e., row 4 of Table 3). The location of the bar whose damage severity is 0.97 indicates the identified location of damage and its extent (3% thickness reduction). Both controllers successfully identified damage from the root to the free end of the beam. However, damage identification based on a noise-free frequency measurement of closed-loop system (A) is consistently accurate in both localization and determining extent of damage, while damage identification based on closed-loop system (B) is occasionally ambiguous in damage localization or inaccurate in the determination of damage extent. Figs. 6 and 7 show the localization results for

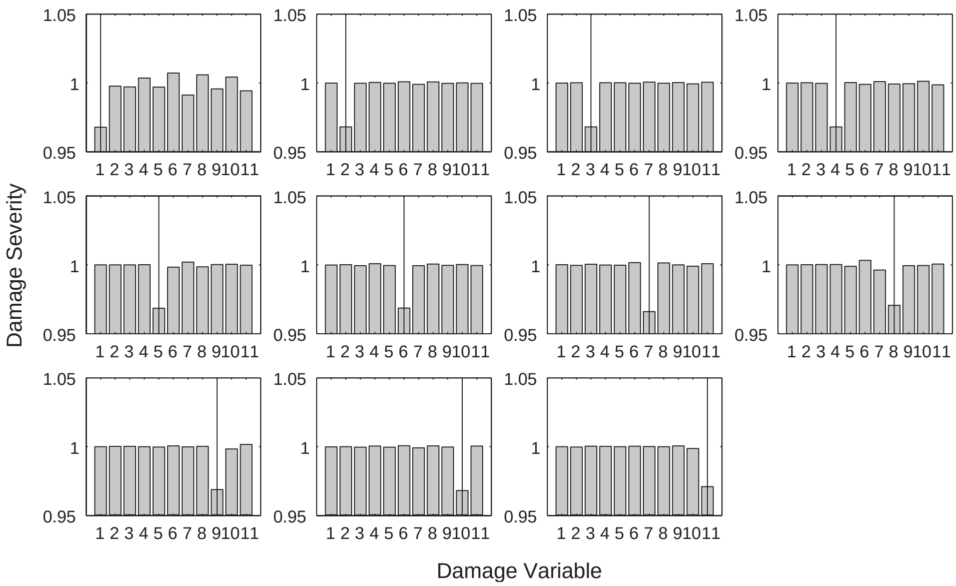

Fig. 4. Localization performance of controller A (*sensitivity enhancement*), with four measured modal frequencies, five closed-loop systems, and 11 different damage cases. Noise-free measured natural frequencies are assumed. Each subfigure corresponds to one of the 11 damage cases. Vertical line denotes the true location of damage.

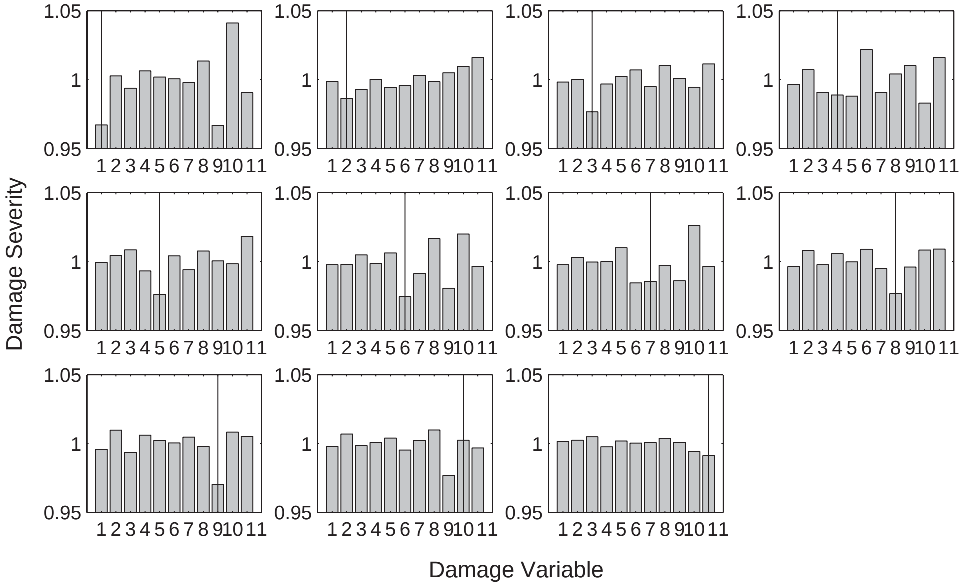

Fig. 5. Localization performance of controller B (*sensitivity reduction*), with four measured modal frequencies, five closed-loop systems, and 11 different damage cases. Noise-free measured natural frequencies are assumed. Vertical line denotes the true location of damage.

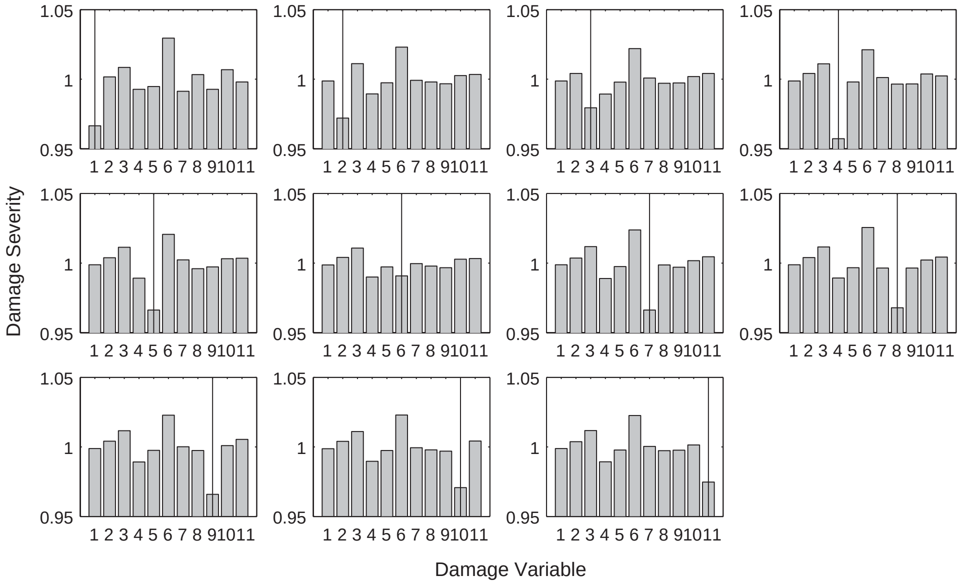

Fig. 6. Localization performance of controller A (*sensitivity enhancement*), with four measured modal frequencies, five closed-loop systems, and 11 different damage cases. Errors of $(-0.03\%, 0.045\%, -0.075\%, 0.06\%)$ for the first four measured natural frequencies are assumed. Vertical line denotes the true location of damage.

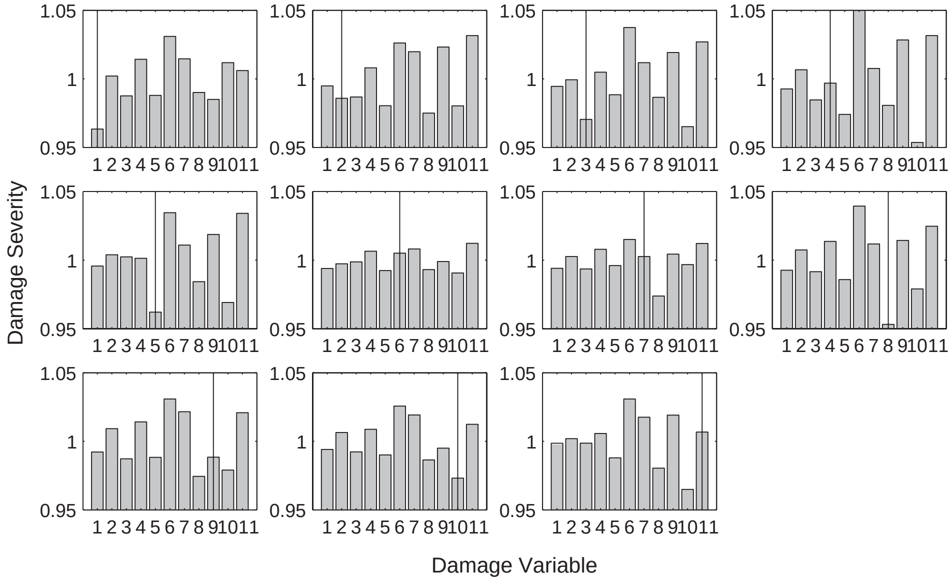

Fig. 7. Localization performance of controller B (*sensitivity reduction*), with four measured modal frequencies, five closed-loop systems, and 11 different damage cases. Errors of $(-0.03\%, 0.045\%, -0.075\%, 0.06\%)$ for the first four measured natural frequencies are assumed. Vertical line denotes the true location of damage.

controllers (A) and (B) with noise-contaminated natural frequency measurements. Here, the measurement noise is assumed as $-0.03\%, 0.045\%, -0.075\%$, and 0.06% error from the first four measured natural frequencies, respectively. In this study, the fixed error ratios for each mode are imposed on measurements so that localization results for each controller can be objectively compared. With measurement noise, damage-sensitive controller (A) still maintains unambiguous localization with modest degradation in determining damage magnitude, while the controller (B), which has no sensitivity enhancement, mostly fails to locate a damaged element for all but four damage locations. Hence, closed-loop systems should have sufficient independence and enhanced sensitivity for damage localization under measurement uncertainties.

In Fig. 8, a comparison is made between the performance in damage localization and the robustness toward measurement noise. The localization performance index, α, is plotted in terms of maximum control effort of the closed-loop system. Maximum control effort is obtained from the impulse response of each closed-loop system. In total, six different closed-loop pole locations are considered: the first one is the closest to the original open-loop pole locations and the last one is the furthest. The horizontal axis is defined as an average of the maximum control effort values from five different closed-loop systems (actuator locations at nodes: 2, 3, 4, 6, and 7) for each closed-loop pole location. Again, a smaller α denotes better ability to localize damage. In general, the magnitude of the maximum control effort depends on how far the closed-loop poles are moved from the original open-loop poles. Noticeable sensitivity enhancement can be achieved if closed-loop poles are assigned further from open-loop poles. Hence, the noise-robustness improves as the effect of sensitivity enhancement surpasses the effect of measurement noise, as shown in the upper line of the figure. As control effort increases, the index, α, decreases, which

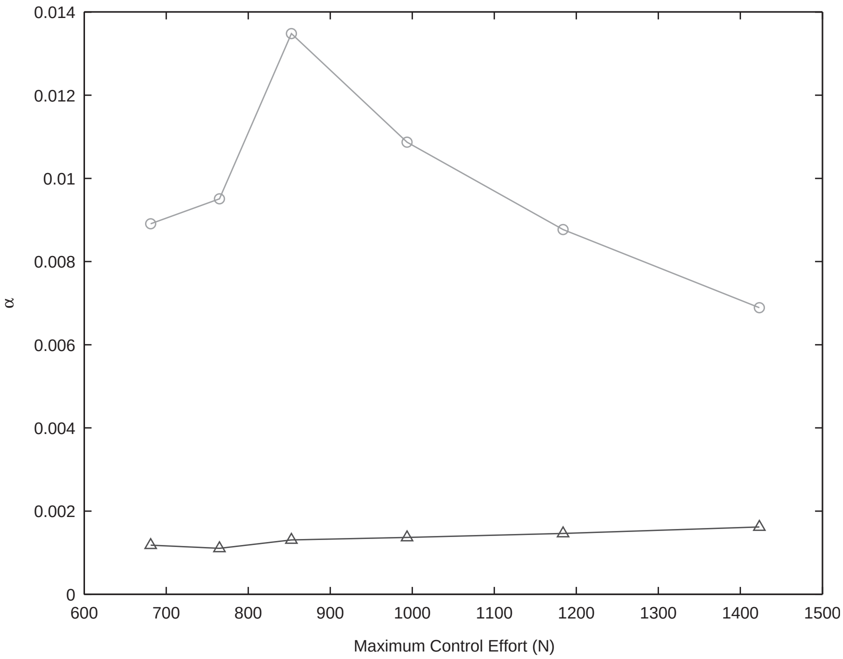

Fig. 8. The average maximum control effort vs. the localization performance index α with measurement noise ($\circ$) and without measurement noise ($\triangle$) in measured frequencies. Closed-loop system (A) with four measured modal frequencies, five closed-loop systems, and 11 damage variables.

means localizing a damaged element becomes more successful. However, the sensitivity enhancement also intensifies the non-linear relation between the damage location and the corresponding modal frequency shift. Thus, the first order approximation becomes less accurate as the maximum control effort increases (lower line). The comparison shows that there is a trade-off between the performance of localization and robustness toward measurement noise. However, from observing the gradients of two lines, sensitivity enhancement is dominant in diminishing the effect of measurement uncertainties.

3.2. *Multi-input controller*

In this section, multi-input closed-loop controllers are developed to complement the single-input closed-loop systems so that the required number of actuators can be minimized. As is well known, the number of identifiable damage variables is limited by the number of measurable modal frequencies and the number of closed-loop systems. For example, a sensitivity matrix having five single-input actuators along with the first three modal frequency measurements cannot identify 16 damage variables ($5 \times 3 < 16$) as shown in Fig. 9.

In order to overcome this problem without increasing the number of actuators, multi-input controllers are developed by coupling single-input actuators, in order to generate additional independent sets of closed-loop systems. In other words, with five single-input actuators, up to 15 different one or two input control laws (five single input and 10 two input) can be developed.

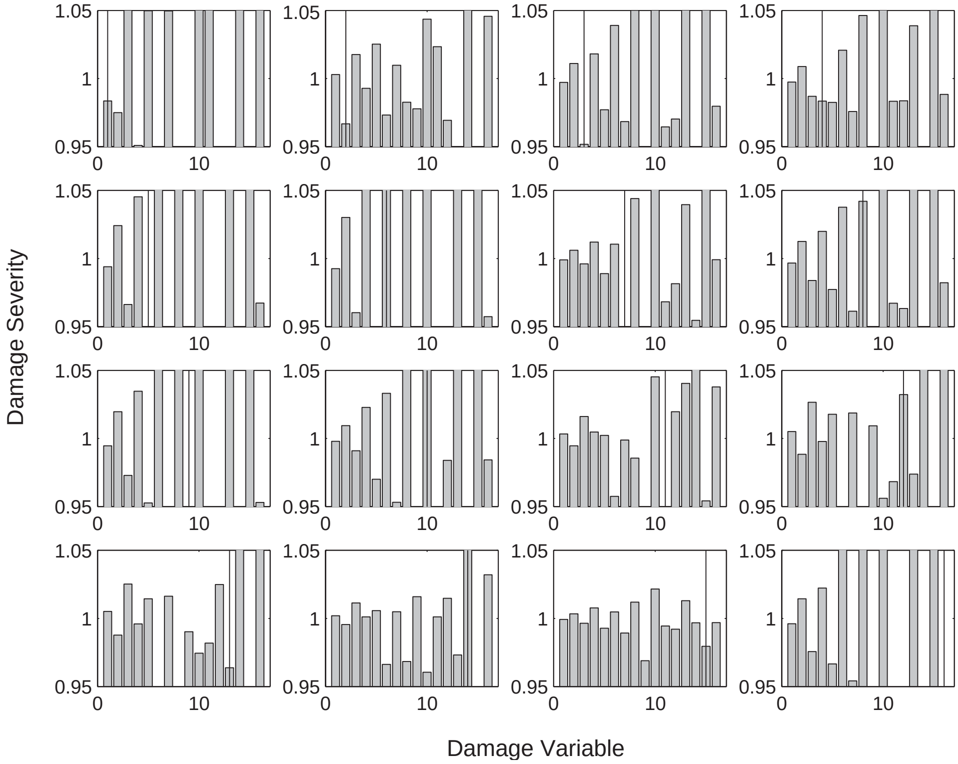

Fig. 9. Localization performance of controller A (*sensitivity enhancement*), with three measured modal frequencies, five closed-loop systems, and 16 different damage cases. Noise free measured natural frequencies are assumed. Vertical line denotes the true location of damage.

Hence, the number of identifiable damage variables can be increased or the minimum required number of actuators can be reduced by incorporating multi-input closed-loop systems. Fig. 10 illustrates the result of damage localization with 10 mixed closed-loop systems, having both single-input and multi-input controllers identified in Table 2. It is apparent from Fig. 10 that the concept of including multi-input controllers can increase the number of identifiable damage variables. The inconsistent result in the first identified damage variable is due to the relatively large frequency shifts for damage at the root of the cantilevered beam. In other words, the first order linear approximation suffers from nonlinearity, as mentioned in Section 3.1. As may be seen from Table 3, the localization performance indices, α and β, decrease noticeably as compared with single-input case with the same number of damage variables, when closed-loop systems with multi-input actuators are included in the sensitivity matrix.

Note that not all of the 15 possible combinations of multi-input closed-loop systems are linearly independent. Here, the pairs of actuators are carefully selected to enhance the independence of closed-loop systems and the condition number of the sensitivity matrix. An open and difficult problem is to find the optimal set of actuator locations and corresponding control gains given the number of damage cases. Ray and Marini [14] investigate the control law optimization problem for a single-input system. In this paper, we show that given an actuator location, scalar metrics α

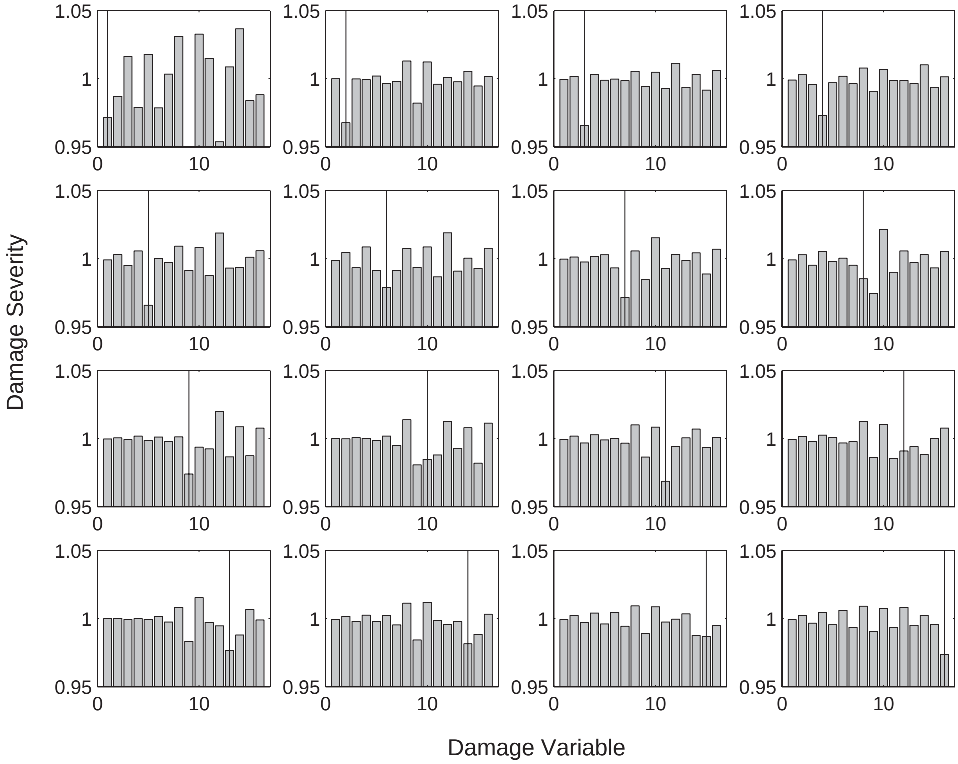

Fig. 10. Localization performance of controller A (*sensitivity enhancement*), with three measured modal frequencies, 10 closed-loop systems, and 16 different damage cases. Noise free measured natural frequencies are assumed. Vertical line denotes the true location of damage.

and β provide measures of damage localization ability, given a set of single- or multi-input actuator locations. Thus, using the optimization method from Ref. [14], along with combinatorics, one can obtain a substantial improvement in numerical conditioning of the damage localization problem.

4. Conclusions

An improved structural damage localization method is presented whose effectiveness is verified by using only measured closed-loop natural frequency changes before and after the damage occurs. Damage-sensitive, or sensitivity enhancing state-feedback controllers are developed to generate multiple independent closed-loop systems. These additional closed-loop modal frequencies provide the eigenvalue sensitivity matrix to approximate damage variables in a least-squares sense. The proposed method is proven to be highly tolerant to measurement noise. Also, the concept of minimum-gain eigenstructure assignment is applied to design multi-input controllers, which improves the condition of the sensitivity matrix with a minimum number of actuators.

References

[1] S.W. Doebling, C.R. Farrar, M.B. Prime, D.W. Shevitz, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review, Los Alamos National Laboratory Technical Report LA-13070-MS, 1996.

[2] H. Gysin, Comparison of expansion methods for FE modeling error localization, *Proceedings of Eighth International Modal Analysis Conference*, 1990, pp. 195–204.

[3] C.S. Lin, Location of modeling errors using modal test data, *American Institute of Aeronautics and Astronautics Journal* 28 (9) (1990) 1650–1654.

[4] E. Dascotte, Practical application of finite element tuning using experimental modal data, *Proceedings of Eighth International Modal Analysis Conference*, 1990, pp. 1032–1037.

[5] M.I. Friswell, J.E.T. Penny, The practical limits of damage detection and location using vibration data, *Proceedings of 11th VPI&SU Symposium on Structural Dynamics and Control*, Blacksburg, VA, 1997, pp. 31–40.

[6] A. Messina, E.J. Williams, T. Contursi, Structural damage detection by a sensitivity and statistical-based method, *Journal of Sound and Vibration* 216 (5) (1998) 791–808.

[7] P.D. Cha, W. Gu, Model updating using an incomplete set of experimental modes, *Journal of Sound and Vibration* 233 (4) (2000) 587–600.

[8] N.G. Nalitolela, J.E.T. Penny, M.I. Friswell, A mass or stiffness addition technique for structural parameter updating, *International Journal of Analytical and Experimental Modal Analysis* 7 (3) (1992) 157–168.

[9] J.-S. Lew, J.-N. Juang, Structural damage detection using virtual passive controllers, *Journal of Guidance, Control, and Dynamics* 25 (3) (2002) 419–424.

[10] A.S.J. Swamidas, Y. Chen, Monitoring crack growth through change of modal parameters, *Journal of Sound and Vibration* 186 (2) (1995) 325–343.

[11] M. Palacz, M. Krawczuk, Vibration parameters for damage detection in structures, *Journal of Sound and Vibration* 249 (5) (2002) 999–1010.

[12] L.R. Ray, L. Tian, Damage detection in smart structures through sensitivity enhancing feedback control, *Journal of Sound and Vibration* 227 (5) (1999) 987–1002.

[13] L.R. Ray, B.H. Koh, L. Tian, Damage detection and vibration control in smart plates: Towards multifunctional smart structures, *Journal of Intelligent Material Systems and Structures* 11 (9) (2000) 725–739.

[14] L.R. Ray, S. Marini, Optimization of control laws for damage detection in smart structures, *Smart structures and materials 2000: Mathematics and Control in Smart Structures, Proceedings of SPIE* Vol. 3984, 2000, pp. 395–402.

[15] J.-N. Juang, K.B. Lim, J.L. Junkins, Robust eigensystem assignment for flexible structures, *Journal of Guidance, Control, and Dynamics* 12 (3) (1989) 381–387.

[16] J.-N. Juang, M.Q. Phan, *Identification and Control of Mechanical Systems*, Cambridge University Press, Cambridge, 2001.